

The Factor-Label Method of Unit Conversion*

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Three Simple Steps

In Engineering disciplines, we use the three-step Factor-Label Method of Unit Conversion to solve algebraic problems with mixed units.

Step 1 Write the algebraic equation so the desired quantity is on the left of the equals sign, and an algebraic expression is on the right of the equals sign.

Step 2 Draw a horizontal line on the page, and enter numbers and units above and below the line according to the algebraic expression.

Step 3 Draw a vertical line to show the separation between each unit conversion, and enter all unit conversions necessary to solve the problem. If the unit is raised to a power, then the conversion factor **and** unit must be raised to that power. Consider memorizing the most common conversion factors, like the ones at the right. See **Appendix A** for more unit conversions and metric prefixes, and **Appendix B** for materials properties such as thermal expansion coefficient and Young's modulus. Table B1 gives these properties in U.S. Customary units, while Table B2 gives them in S.I. units.

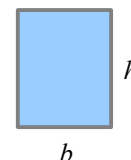
The units in the final answer **must** appear in the equation, **and** all other units must cancel.

1 ft.	=	12 in.	
1 m	=	100 cm	= 10 ³ mm
1 kip	=	10 ³ lb.	
1 Pa	=	1 N/m ²	
1 N	=	1 kg·m/s ²	

Metric prefixes			
milli-	(m)	=	10 ⁻³
centi-	(c)	=	10 ⁻²
kilo-	(k)	=	10 ³
Mega-	(M)	=	10 ⁶
Giga-	(G)	=	10 ⁹

Example #1

The area of a rectangle is $A=b \cdot h$. Given a base $b=83$ in. and a height $h=45$ ft., calculate the area in square feet.



Step 1 The algebraic equation does not need to be manipulated.

Step 2 Draw a horizontal line. Enter 83 in. and 45 ft. in the numerator.

$$A = b \cdot h = \frac{83 \text{ in.} \cdot 45 \text{ ft.}}{\quad}$$

Step 3 We want to eliminate inches to obtain a final result in square feet. Therefore, put 12 inches in the denominator of the unit conversion, and 1 ft. in the numerator.

$$A = b \cdot h = \frac{83 \text{ in.} \cdot 45 \text{ ft.}}{12 \text{ in.}} \left| \frac{\text{ft.}}{12 \text{ in.}} \right. = 311.25 \text{ ft.}^2$$

* Adapted from Barry Dupen, *Applied Strength of Materials for Engineering Technology*, v. 11, January 2016, available at http://www.etc.s.ipfw.edu/~dupenb/ET_200/StrengthOfMatHome.html.

Example #2

Stress is force divided by area. If the stress is 1 N/mm², what is the stress in MPa?

Step 1 There is no algebra to solve here because we are converting one unit to another.

Step 2 Draw a horizontal line. Enter 1 N in the numerator, and mm² in the denominator.

$$\frac{1 \text{ N}}{\text{mm}^2}$$

Step 3 A pascal is defined as Pa=N/m², so enter Pa in the numerator. Instead of writing in N/m² the denominator, put N in the denominator and m² in the numerator.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right.$$

Now enter the unit conversions to eliminate the two area terms: m² and mm². There are 10³ mm in a meter, so use parentheses to square the number and the unit.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(10^3 \text{ mm})^2}{\text{m}^2}$$

Finally, 1MPa=10⁶Pa. Put MPa in the numerator and 10⁶ Pa in the denominator. If you write the equation without numbers, it looks like $\frac{\text{N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{\text{mm}^2}{\text{m}^2} \left| \frac{\text{MPa}}{\text{Pa}} \right.$. Cross out duplicate terms, and all terms cancel except for MPa. If you write the equation without units, it looks like $\frac{1}{10^6}$.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(10^3 \text{ mm})^2}{\text{m}^2} \left| \frac{\text{MPa}}{10^6 \text{ Pa}} \right.$$

$$\frac{\text{N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(\text{mm})^2}{\text{m}^2} \left| \frac{\text{MPa}}{\text{Pa}} \right.$$

Solving the equation with numbers and units, we get 1 N/mm²=1 MPa . This is a useful conversion factor in many SI engineering problems.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(10^3 \text{ mm})^2}{\text{m}^2} \left| \frac{\text{MPa}}{10^6 \text{ Pa}} \right. = 1 \text{ MPa}$$

Example #3

Deflection due to thermal expansion is $\delta = \alpha \cdot L \cdot \Delta T$. The upper-case Greek letter delta means “change”, so ΔT means “change in temperature.” Given a deflection $\delta = 0.06 \text{ in.}$, a length $L = 8 \text{ ft.}$, and a thermal expansion coefficient $\alpha = 5 \times 10^{-6} \text{ in./in.}^\circ\text{F}$, calculate the change in temperature in degrees Fahrenheit.



Step 1 Rewrite the equation algebraically to solve for ΔT .

$$\delta = \alpha \cdot L \cdot \Delta T \rightarrow \Delta T = \frac{\delta}{\alpha L}$$

Step 2 Draw a horizontal line. Enter 0.06 in. in the numerator. Since the units for α are a fraction, enter $5 \times 10^6 \text{ in.}$ in the denominator and in.^oF in the numerator. Enter 8 ft. in the denominator.

$$\Delta T = \frac{0.06 \text{ in.}}{5 \times 10^{-6} \text{ in.}} \frac{\text{in.}^\circ\text{F}}{8 \text{ ft.}}$$

Step 3 Convert feet to 12 inches so the length units to cancel, and the result is in ^oF.

$$\Delta T = \frac{0.06 \text{ in.}}{5 \times 10^{-6} \text{ in.}} \frac{\text{in.}^\circ\text{F}}{8 \text{ ft.}} \left| \frac{\text{ft}}{12 \text{ in.}} \right. = 125 \text{ }^\circ\text{F}$$

Example #4

Stress is force divided by area: $\sigma = \frac{P}{A}$. Given a force $P = 7000 \text{ lb.}$ acting on an area $A = 3 \text{ ft.}^2$, calculate the stress in units of pounds per square inch (psi).

Step 1 The equation does not need to be manipulated.

Step 2 Draw a horizontal line. Enter 7000 lb. in the numerator, and 3 ft.² in the denominator.

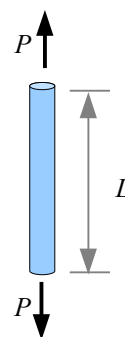
$$\sigma = \frac{P}{A} = \frac{7000 \text{ lb.}}{3 \text{ ft.}^2}$$

Step 3 The stress is in units of pounds per square foot. There are 12 inches in a foot, but we need to convert square feet, so square the number and the unit: $(12 \text{ in.})^2$. Square feet cancel, and the answer is in pounds per square inch, also written psi.

$$\sigma = \frac{P}{A} = \frac{7000 \text{ lb.}}{3 \text{ ft.}^2} \left| \frac{\text{ft.}^2}{(12 \text{ in.})^2} \right| = 16.2 \frac{\text{lb.}}{\text{in.}^2} = 16.2 \text{ psi}$$

Example #5

A tensile bar stretches an amount $\delta = \frac{P \cdot L}{A \cdot E}$ where P is the applied load, L is the length of the bar, A is the cross-sectional area, and E is Young's Modulus. The bar has a circular cross section. Given a load of 30 kN, a length of 80 cm, a diameter of 6 mm, and a Young's Modulus of 207 GPa, calculate the deflection in mm.



Step 1 In math class, the area of a circle is given by $A = \pi r^2$. In real life, we measure diameter using calipers; it is much easier to measure a diameter than a radius on most objects. Convert radius to diameter, and the area equation becomes more useful. This is a good equation to memorize.

$$A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2 = \frac{\pi d^2}{4} = \frac{\pi}{4} d^2$$

Combine the two equations to obtain a single algebraic equation.

$$\delta = \frac{P \cdot L}{A \cdot E} = \frac{4 P L}{\pi d^2 E}$$

Step 2 Draw a horizontal line and enter the numbers and units.

$$\delta = \frac{4 \cdot 30 \text{ kN} \cdot 80 \text{ cm}}{\pi (6 \text{ mm})^2 207 \text{ GPa}}$$

Step 3 The SI unit of stress or pressure is the pascal, where

$\text{Pa} = \frac{\text{N}}{\text{m}^2}$, so $\text{GPa} = \frac{10^9 \text{ N}}{\text{m}^2}$. Since $1 \text{ kN} = 10^3 \text{ N}$, we can write $\text{GPa} = \frac{10^6 \text{ kN}}{\text{m}^2}$. Three conversion factors are needed:

one to cancel GPa and kN; a second to cancel mm² and m²; and a third to put the final answer in mm.

$$\delta = \frac{4 \cdot 30 \text{ kN} \cdot 80 \text{ cm}}{\pi (6 \text{ mm})^2 207 \text{ GPa}} \left| \frac{\text{GPa} \cdot \text{m}^2}{10^6 \text{ kN}} \right| \left| \frac{(10^3 \text{ mm})^2}{\text{m}^2} \right| \left| \frac{10 \text{ mm}}{\text{cm}} \right| = 4.1 \text{ mm}$$

Example #6

The weight of a solid object is the specific weight of the material times the volume of the object: $W = \gamma V$. The volume of a rod, pipe, or bar is the cross-sectional area times the length: $V = AL$. Given the specific weight of steel is 0.284 lb./in.^3 , calculate the weight of a 2 inch diameter, 3 foot long bar of steel.

Step 1 Combine the two equations to solve for weight: $W = \gamma AL$. Since the rod is round, the cross-sectional area is

$$A = \frac{\pi}{4} d^2, \text{ therefore } W = \frac{\gamma \pi d^2 L}{4}$$

Step 2 Draw a horizontal line and enter the numbers and units.

$$W = \frac{\gamma \pi d^2 L}{4} = \frac{0.284 \text{ lb.}}{\text{in.}^3} \cdot \frac{\pi (2 \text{ in.})^2 3 \text{ ft.}}{4}$$

Step 3 The only unit conversion is feet to inches.

$$W = \frac{\gamma \pi d^2 L}{4} = \frac{0.284 \text{ lb.}}{\text{in.}^3} \cdot \frac{\pi (2 \text{ in.})^2 3 \text{ ft.}}{4} \left| \frac{12 \text{ in.}}{\text{ft.}} \right. = 32.1 \text{ lb.}$$

Example #7

Calculate the weight of a 5 cm diameter, 2 meter long bar of steel, given the density of steel is 7.85 g/cm^3

Step 1 Use $W = \frac{\gamma \pi d^2 L}{4}$ from Example #6. Specific weight is density times gravity: $\gamma = \rho g$, so $W = \frac{\rho g \pi d^2 L}{4}$.

Step 2 Draw a horizontal line and enter the numbers and units.

$$W = \frac{\rho g \pi d^2 L}{4} = \frac{7.85 \text{ g}}{\text{cm}^3} \cdot \frac{9.81 \text{ m}}{\text{s}^2} \cdot \frac{\pi (5 \text{ cm})^2 2 \text{ m}}{4}$$

Step 3 The SI unit of weight is the newton: $\text{N} = \frac{\text{kg m}}{\text{s}^2}$.

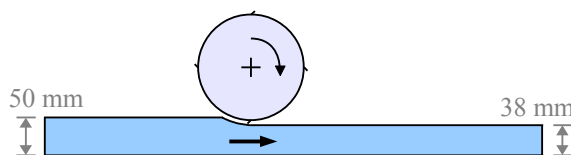
$$W = \frac{7.85 \text{ g}}{\text{cm}^3} \cdot \frac{9.81 \text{ m}}{\text{s}^2} \cdot \frac{\pi (5 \text{ cm})^2 2 \text{ m}}{4} \left| \frac{\text{kg}}{10^3 \text{ g}} \right| \left| \frac{100 \text{ cm}}{\text{m}} \right| \left| \frac{\text{N s}^2}{\text{kg m}} \right. = 302 \text{ N}$$

Notice the unit “g” for grams and the term “g” for gravity. In science and engineering, we tend to use roman type for units, and *italic type* for variables. Another example is a block sliding on an inclined plane, where “N” stands for newtons and “N” stands for normal force.

Example #8

A 50 mm thick wood board is planed to a thickness of 38 mm. Calculate how much material was removed, in percent.

Calculate the percent change by subtracting the initial value from the final value, then dividing by the initial value.



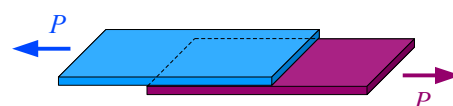
This method works whether you are calculating thickness change, weight change, price change, or any other kind of change. The word “percent” means “per hundred”, so a result of 0.36 is 36%.

$$\frac{t_f - t_o}{t_o} = \frac{38 \text{ mm} - 50 \text{ mm}}{50 \text{ mm}} = -0.24 \text{ or } -24\% \quad \text{The minus sign means the value decreased.}$$

Example #9

Two pieces of carbon fiber composite are glued together as a lap joint.

Shear stress is $\tau = \frac{P}{A}$, where A is the overlapped area.



If the glue has a shear strength of 1.2 ksi and the joint is loaded with a shear load $P = 1700 \text{ lb.}$, how much glued surface area is required for a good bond?

Solve algebraically, and split ksi into $\frac{\text{kips}}{\text{in.}^2}$. Surface area $A = \frac{P}{\tau} = \frac{1700 \text{ lb.}}{1.2 \text{ kips}} \cdot \frac{\text{in.}^2}{1000 \text{ lb.}} = 1.42 \text{ in.}^2$

Sample Problems

Write an algebraic equation first, then introduce numbers and units, using the Factor-Label Method of Unit Conversion. Report the answer in the units given in brackets.

Unit conversions: $1000 \text{ L} = 1 \text{ m}^3$, $100 \text{ cm} = 1 \text{ m}$, $\rho_{\text{water}} = 1000 \text{ kg/m}^3$, $\gamma_{\text{water}} = 62.4 \text{ lb./ft.}^3$, $231 \text{ in.}^3 = 1 \text{ gal.}$, $12 \text{ in.} = 1 \text{ ft.}$, $5280 \text{ ft.} = 1 \text{ mile}$, $g = 32.2 \text{ ft./s}^2 = 9.81 \text{ m/s}^2$.

1. A $\frac{1}{2}$ mile length of polyethylene water pipe is installed at an organic parsley farm. The inside diameter of the pipe is 1.816 in. For cleanliness, the pipe has to be flushed with twice its volume of water before the water is safe to drink. How much water is required for a complete system flush? [gal.]
2. How much does this water weigh? [lb.]
3. What is the volume of a sphere having a diameter of 264 cm? [m^3]
4. What is the mass of 42,000 L of water? [kg]
5. What is the weight of 42,000 L of water? [N]
6. What is the diameter of a circle having an area of 4 ft.^2 ? [in.]